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MAXIMAL AND MINIMAL IDEALS IN WEAKLY STANDARD RINGS

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ABSTRACT

We prove that

- If the right ideal A of a weakly standard ring R is maximal and nil, A is a two – sided ideal and
- If A is minimal, then it is either a two – sided ideal of R or the ideal it generates is contained in the nucleus.

KEYWORDS: Alternative Ring, Characteristic Ring, Maximal, Weakly Standard Ring, Nucleus

INTRODUCTION

Hentzel and Smith [1] studied the properties of ideals of right alternative ring R with characteristic $\neq 2$. They proved that if a left ideal L of R is maximal and nil, then L is a two-sided ideal and that when L is minimal, then it is either a two sided ideal or the ideal it generates is contained in the right nucleus of R. Also they have constructed the ideals in locally (-1, 1) rings with characteristic $\neq 2$. Paul [2] studied some properties of ideals in accessible rings and in anti flexible rings. In this paper we prove similar properties of maximal and minimal right ideals in weakly standard rings.

Preliminaries

Throughout this paper R denotes a weakly standard ring. In a weakly standard ring we have the flexible identity $(x, y, x) = 0$ and the following identities:

$((w, x), y, z) = 0$ 1

and $(w, (x, y), z) = 0.$ 2

In any ring $(wx, y, z) - (w, xy, z) + (w, x, yz) = (w, x, y) z + w(x, y, z).$ 3

We know that a linearization of the flexible law yields the identity

$(x, y, z) + (z, y, x) = 0$ 4

An expression of the form (R, a, b) means the set of all finite sums (x, a, b) for $x \in R$; analogous arguments are meant for other form of similar expressions. The nucleus N of R is the set of all elements n in R such that $(n, R, R) = (R, n, R) = (R, R, n) = 0$. An ideal A in R is maximal (or minimal) if $A \neq R$ and for any ideal $B \supseteq A$ (or $B \subseteq A$), either B = A or B = R. An ideal A in R is a nil ideal if each element of A is nilpotent.

By using $s (x, y, z) = 0$ 5

of characteristic $\neq 2$, we have proved that

$(x, y, z) + (y, z, x) + (z, x, y) = 0.$ 6

Now we prove the following properties of the maximal and minimal ideals of R.

MAIN RESULTS

Theorem 1: Let R be a prime weakly standard ring and let A be a right ideal of R.

- If A is maximal and nil, then A is a two-sided ideal of R.
- If A is minimal, then it is either a two-sided ideal of R or the ideal it generates is contained in the nucleus.

Proof: (i) Suppose the right ideal A is maximal and nil. If $aA \not\subset R$ for some $a \in R$, we consider $A + aA$. This is a right ideal, since using (6), we have

$(a A) R \subseteq (a, A, R) + a (AR)$

$\subseteq (A, a, R) – (A, R, R) + aA$

$\subseteq A + aA.$

Thus $A \not\subset A + aA$ and A maximal imply $R = A + aA$.

Let $a = x_1 + ax_2$ where $x_1, x_2 \in A$. Then iterations for a in the right side of this equation, give $a = x_3 + (((ax_2) x_{2)}\ldots x_{2)} x_2)$, where $x_3 \in A$ and x_2 is a factor n times. Now $(R, A, A) \subseteq (A, A, R) \subseteq A$ by (4), and so by finite induction we see that $a = x_4 + a(x_2)^n$ where $x_4 \in A$. But since A is nil, $(x_2)^n = 0$ for some n. Thus $a \in R$, which means $aA \subseteq A$ is a contradiction. We therefore have $aA \subseteq A$ for $a \in R$, i.e. A is a two-sided ideal of R.

(ii) Let us next assume that the right ideal A is minimal, but not a two-sided ideal. Then there exists an $a\in R$ such that $aA \not\subset A$. Let $A^1 = \{x \in A; ax \in A\}$. Now by using (5), $x \in A^1$ implies $xr \in A$ and $a(xr) = x(ar) + (ax – xa)r + (xr)a - x(ra) \in A^1$ for all $r\in R$. Thus it follows $A^1 \subset A$ is a right ideal, and so by the minimality of A we have $A^1 = (0)$. Clearly $(A, R, R) \subseteq A$.

By (3), $a (r, x, y) = (ar, x, y) - (a, rx, y) + (a, r, xy) - (a, r, x) y$

By (1), $(ar, x, y) = (ra, x, a)$ and by (6).

$(a, rx, y) = -(rx, y, a) + (rx, a, y),$

$(a, r, xy) = -(r, xy, a) + (r, a, xy),$

$(a, r, x) = -(r, x, a) + (r, a, x).$

Thus $a (r, x, y) = (ra, x, y) + (rx, y, a) - (rx, a, y) - (r, xy, a) + (r, a, xy) + (r, x, a) y - (r, a, x) y.$

Hence $a(r, x, y)\in A$. This implies that $(A, R, R) \subseteq A^| = (0)$, i.e., $A \subseteq N$. We set $W_0 = A$ and $W_{i+1}=W_i+R W_i$ for $i \geq 0$. Suppose W_i is a right ideal of R and contained in N.

Then $W_{i+1} R = (W_i +RW_i)R \subseteq W_i + (RW_i)R$

$\subseteq \quad W_i + R (W_i R)$

$\subseteq \quad W_i + RW_i = W_{i+1}$,

i.e., W_{i+1} is a right ideal. Using (1) $W_i \subseteq N$, we have

$(W_{i+1}, R, R) = (W_i, R, R) + (RW_i, R, R)$

$= (W_iR, R, R)$

$\subseteq (W_i, R, R)$

$= (0)$,

i.e., $W_{i+1} \subseteq N$.

Thus it follows by induction that W_i is a right ideal contained in N. Since the ideal generated in R by A is simply $\bigcup_{i=0}^{\infty} W_i$. This completes the proof.

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